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Extension Research

Designing Control Charts for Minimum Total Quality Costs

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Abstract

The basic control chart consists of sampling from a process over time and charting process measurements. For as long as the measurements fall inside control limits (L), the process is considered in-control. In feed production, it is customary to set the control limits at two standard deviations. This practice does not consider the probability of both Type I and Type II errors in addition to all the costs incurred. The objective of this research was to derive a general methodology to determine optimum sample size (n), sampling period (h), and location of L for an X chart used to monitor production processes. A quality cycle is defined as the time between the start of successive in-control periods. Let A_1 be the average production length while in-control and A_2 when the process has shifted to an out-of-control state. The total cost of quality is given by $C = \{C_0/\lambda + C_1 (-t + nE + h(A2) + \delta_1 T_1 + \delta_2 T_2) + sY/A_1 + W\} \div \{1/\lambda + (1 - \delta_1)sT_0/A_1 - t + nE + h(A_2) + T_1 + T_2\} + \{[a + bn)/h] \times [1/\lambda - t + nE + h(A_2) + \delta_1 T_1 + \delta_2 T_2]\} \div \{1/\lambda + (1 \ \delta_1)sT_0/A_1 - t + nE + h(A_2) + T_1 + T_2\}$, with $s = \exp(-\lambda h) / (1 - \exp(-\lambda h))$, $t = [1 - 1 + \lambda h)\exp(-\lambda h)] / [\lambda(1 - \exp(-\lambda h)]$, $A_1 = 1/\alpha$, $A_2 = 1/(1 - \beta)$, $\alpha = \Pr(X - CL > L\sigma/\sqrt{n}) + \Pr(X \ - CL < -L\sigma/\sqrt{n} < X - CL < L\sigma/\sqrt{n})$, where t = expected time of occurrence of the assignable cause; s = expected number of samples taken while in-control; λ = 1/mean time process is in-control; E = time to sample and chart one item; T_0 = expected search time when false alarm; T_1 = expected time to discover the assignable cause; T_2 = expected time to repair the process; δ_1 = 1 if production continues during searches, 0 otherwise; δ_2 = 1 if production continues during repair, 0 otherwise; C_0 = quality cost/hour while producing in-control; C_1 = quality cost/hour while producing out-of-control; Y = cost per false alarm; W = cost to locate and repair the assignable cause; a = fixed cost per sample; b = cost per unit sampled. The values n*, h* and L* that give the minimum C provides the optimum control design. The procedure is easily implemented on a modern spreadsheet.

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